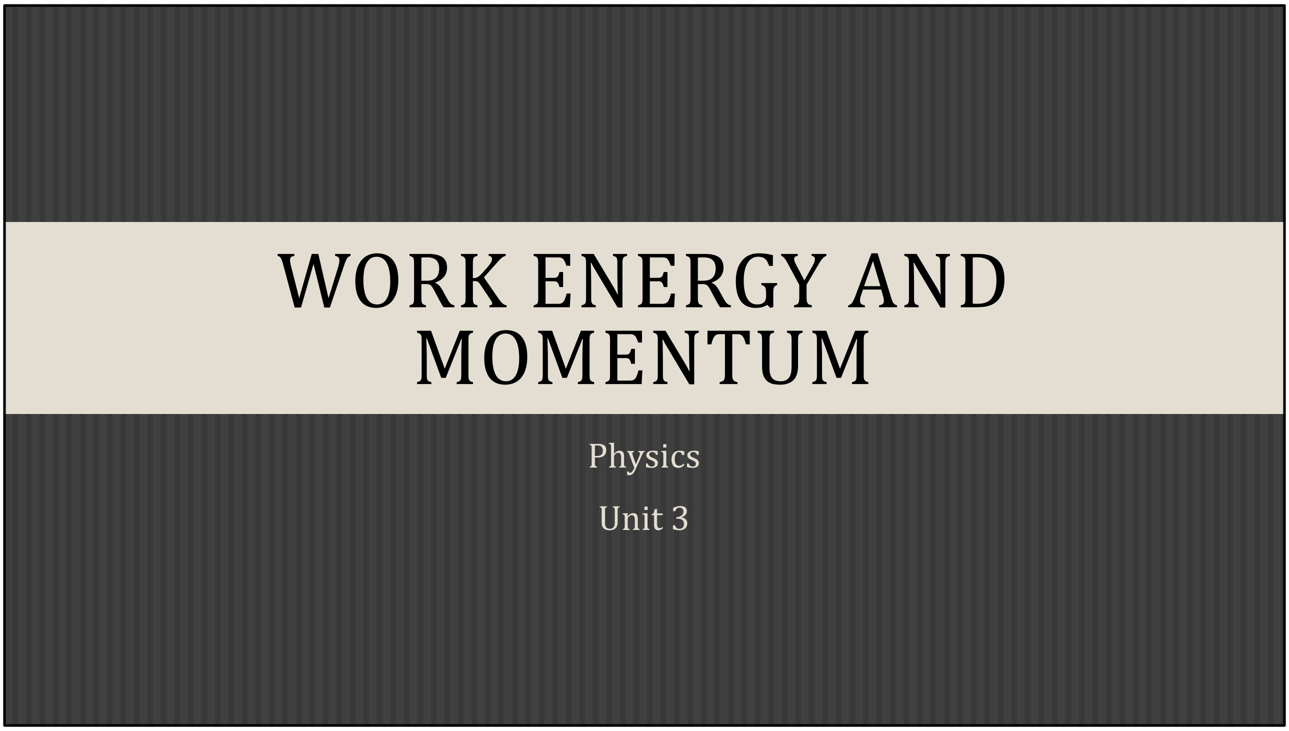

WORK ENERGY AND MOMENTUM

Physics

Unit 3

Physics Unit 3

- This Slideshow was developed to accompany the textbook
  - *OpenStax Physics*
    - *Available for free at https://openstaxcollege.org/textbooks/college-physics*
  - *By OpenStax College and Rice University*
  - *2013 edition*
- Some examples and diagrams are taken from the textbook.

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03-01 WORK AND THE WORK-ENERGY THEOREM

In this lesson you will…

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.
- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

03-01 WORK AND THE WORK-ENERGY THEOREM

- Which of the following is NOT work?
  - Pushing a Stalled Car
  - Pulling a Wagon
  - Climbing stairs
  - Falling Down
  - [✓ Carrying a Heavy Backpack Down the Hall]

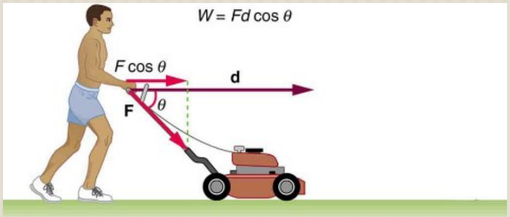
03-01 WORK AND THE WORK-ENERGY THEOREM

- Work
  - Depends on the force and the distance the force moves the object.
  - Want the force in the direction of the distance

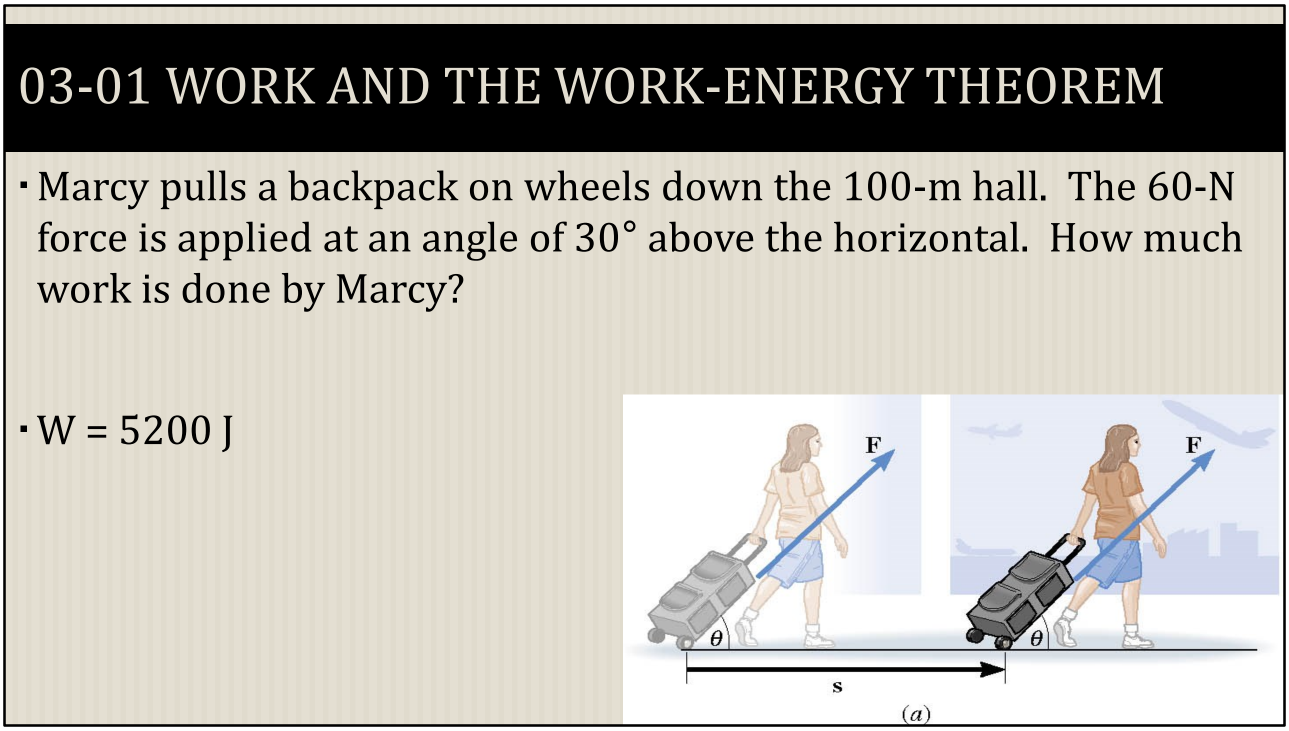
$$W = \vec{F} \cdot \vec{d}$$

$$W = Fd\cos\theta$$

- Unit: $N \cdot m = J$ (Joule) (Scalar)
- Watch Eureka! 08



$$J = \frac{kg \cdot m^2}{s^2}$$



03-01 WORK AND THE WORK-ENERGY THEOREM

- Marcy pulls a backpack on wheels down the 100-m hall. The 60-N force is applied at an angle of 30° above the horizontal. How much work is done by Marcy?

- W = 5200 J

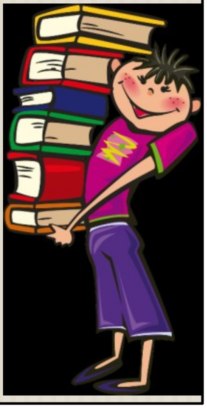
$$W = Fd\cos\theta$$
$$W = (60\ N)(100\ m)\cos 30° = 5196\ J$$

03-01 WORK AND THE WORK-ENERGY THEOREM

- Drew is carrying books (200 N) down the 100-m hall. How much work is Drew doing on the books?

- W = 0 J

- The force is vertical displacement is horizontal.



$$W = Fd\cos\theta$$
$$W = (200\ N)(100\ m)\cos 90° = 0\ J$$

03-01 WORK AND THE WORK-ENERGY THEOREM

- You carry some books (200 N) while walking down stairs height 2 m and length 3 m. How much work do you do?
- W = -400 J

F = 200 N (lift up)
d = 2 m (down)

$$W = Fd\cos\theta = (200\ N)(2\ m)\cos 180° = -400\ J$$

03-01 WORK AND THE WORK-ENERGY THEOREM

- A suitcase is hanging straight down from your hand as you ride an escalator. Your hand exerts a force on the suitcase, and this force does work. Which one of the following is correct?
- The W is negative when you ride up and positive when you ride down
- [✓ The W is positive when you ride up and negative when you ride down]
- The W is positive
- The W is negative

03-01 WORK AND THE WORK-ENERGY THEOREM

- Do work means → $W = Fd$
  - $F = ma$

- So work by a net force gives an object some acceleration

- Acceleration means the velocity changes

03-01 WORK AND THE WORK-ENERGY THEOREM

- $F = ma$
- $Fd = mad$
- ${v_f}^2 = {v_0}^2 + 2ad$ → solve for ad
  - $ad = \frac{1}{2}\left(v_f^2 - v_0^2\right)$
- $Fd = m\frac{1}{2}\left(v_f^2 - v_0^2\right)$

- $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

03-01 WORK AND THE WORK-ENERGY THEOREM

- Energy is the ability to do work
- Kinetic Energy - Energy due to motion
  - If something in motion hits an object, it will move it some distance
- $KE = \frac{1}{2}mv^2$
  - Scalar
  - Unit is joule (J)
  - Watch Eureka! 09

03-01 WORK AND THE WORK-ENERGY THEOREM

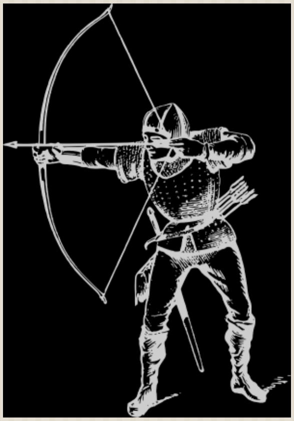
- **Work Energy Theorem**
- **Work of Net external force** = change in kinetic energy

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

$$W = KE_f - KE_0$$

03-01 WORK AND THE WORK-ENERGY THEOREM

- A 0.075-kg arrow is fired horizontally. The bowstring exerts a force on the arrow over a distance of 0.90 m. The arrow leaves the bow at 40 m/s. What average force does the bow apply to arrow?



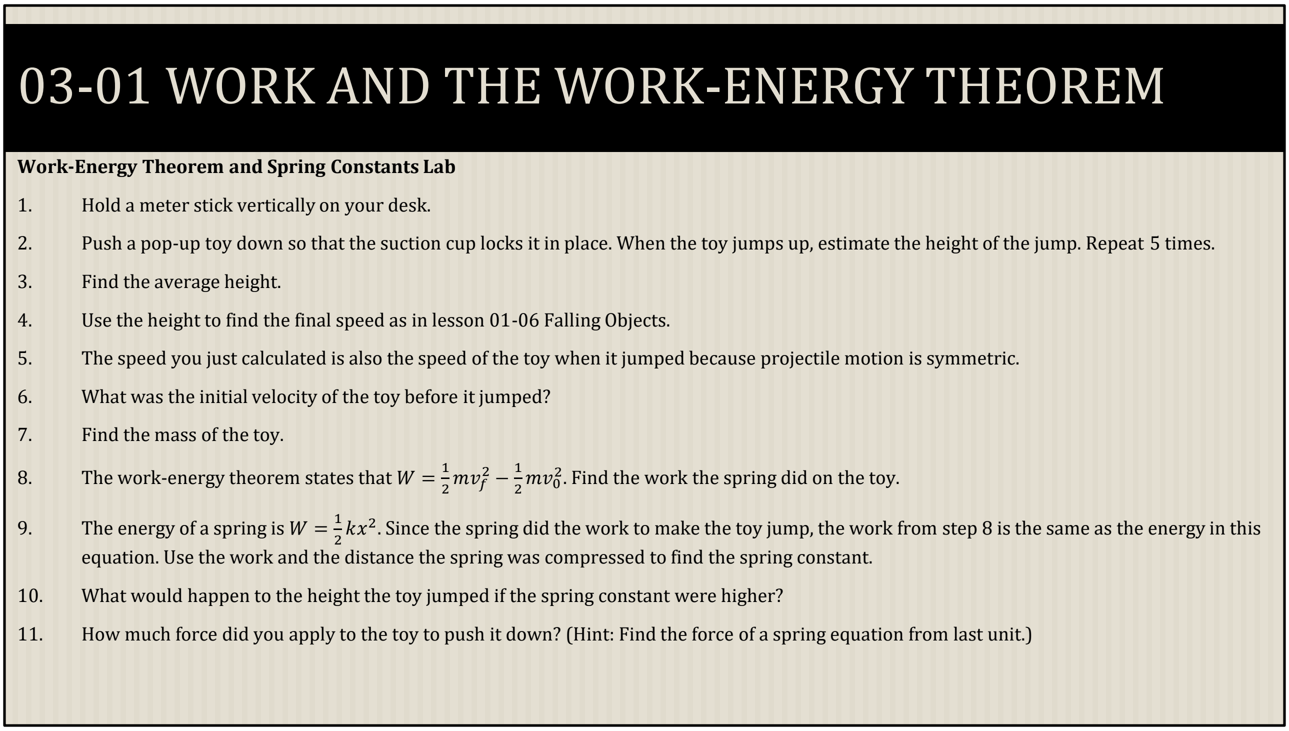
$$m = 0.075\ kg, \qquad s = 0.90\ m, \qquad v_0 = 0\frac{m}{s}, \qquad v_f = 40\frac{m}{s}$$

$$W = KE_f - KE_0$$

$$F(0.90\ m) = \frac{1}{2}(0.075\ kg)\left(40\frac{m}{s}\right)^2 - \frac{1}{2}(0.075\ kg)\left(0\frac{m}{s}\right)^2$$

$$F(0.90\ m) = 60\ J$$

$$F = 66.7\ N$$



03-01 WORK AND THE WORK-ENERGY THEOREM

Work-Energy Theorem and Spring Constants Lab

1. Hold a meter stick vertically on your desk.
2. Push a pop-up toy down so that the suction cup locks it in place. When the toy jumps up, estimate the height of the jump. Repeat 5 times.
3. Find the average height.
4. Use the height to find the final speed as in lesson 01-06 Falling Objects.
5. The speed you just calculated is also the speed of the toy when it jumped because projectile motion is symmetric.
6. What was the initial velocity of the toy before it jumped?
7. Find the mass of the toy.
8. The work-energy theorem states that $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$. Find the work the spring did on the toy.
9. The energy of a spring is $W = \frac{1}{2}kx^2$. Since the spring did the work to make the toy jump, the work from step 8 is the same as the energy in this equation. Use the work and the distance the spring was compressed to find the spring constant.
10. What would happen to the height the toy jumped if the spring constant were higher?
11. How much force did you apply to the toy to push it down? (Hint: Find the force of a spring equation from last unit.)

4. Use $v^2 = v_0^2 + 2a(x - x_0)$ which becomes $v^2 = 2gh$
6. 0
10. It would jump higher

03-01 HOMEWORK

- Do lots of work.

- Read 7.3, 7.4

03-02 POTENTIAL ENERGY AND CONSERVATIVE FORCES

In this lesson you will…

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass m at height h on Earth is given by PEg = mgh.
- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke’s law applies.
- Use the work-energy theorem to show how having only conservative forces implies conservation of mechanical energy.

03-02 POTENTIAL ENERGY AND CONSERVATIVE FORCES

- Do the lab in your worksheet.

- What does this tell you about the PE and KE when objects fall?

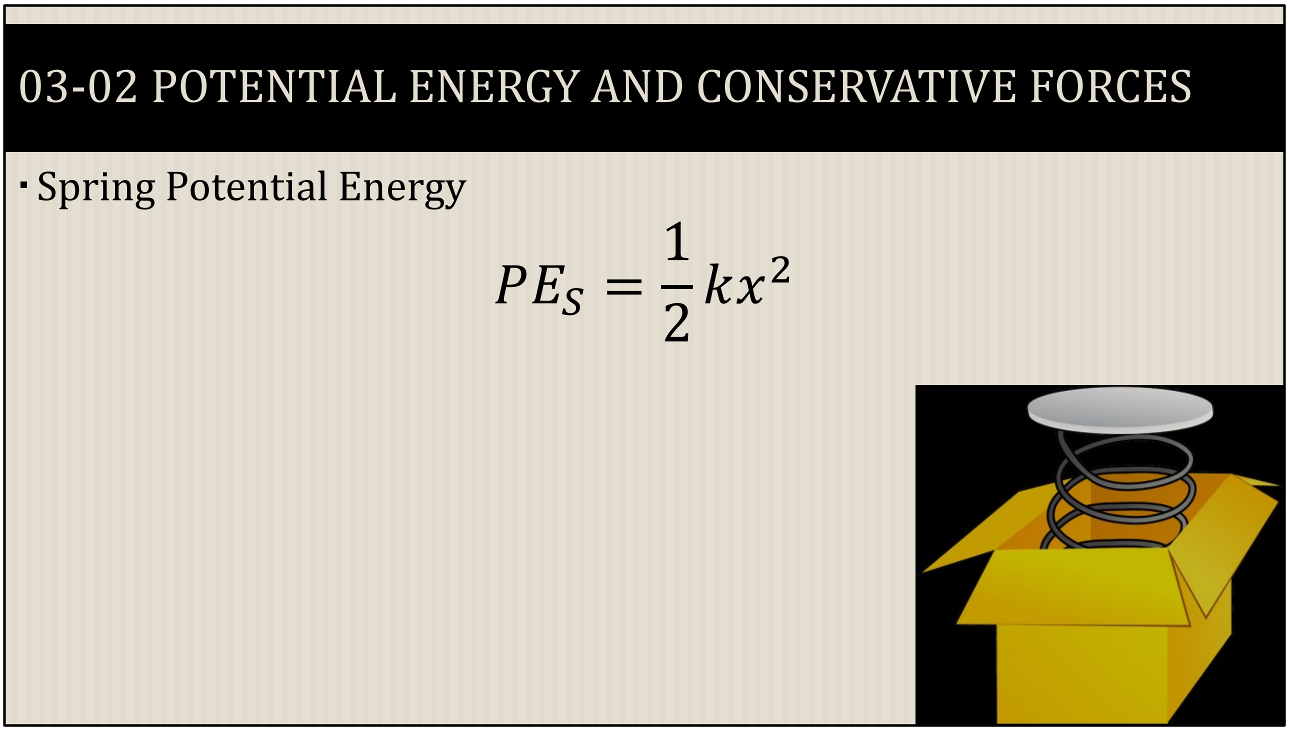
PE changes to KE

03-02 POTENTIAL ENERGY AND CONSERVATIVE FORCES

- Potential energy
  - Energy due to position
- $W = Fd$
- Gravity
- $W_{gravity} = mgh$

$$PE = mgh$$

- Since the force of gravity is down
  - We only worry about the vertical distance
- Potential Energy is not absolute
  - It is a difference
- The path the object takes doesn't matter, just the vertical distance
- h is measured from any chosen point. Just be consistent



03-02 POTENTIAL ENERGY AND CONSERVATIVE FORCES

- Spring Potential Energy

$$PE_S = \frac{1}{2}kx^2$$

$$W = Fd$$

$$PE_S = (average\ from\ 0\ to\ kx\ by\ Hooke's\ Law)x$$

$$PE_S = \frac{1}{2}kx \cdot x$$

$$PE_S = \frac{1}{2}kx^2$$

03-02 POTENTIAL ENERGY AND CONSERVATIVE FORCES

- Watch Eureka! 10
- Conservative Forces
  - A force where the work it does is independent of the path
    - Only thing that matters is starting and stopping point

03-02 POTENTIAL ENERGY AND CONSERVATIVE FORCES

- Examples of conservative forces
  - Gravitational Force
  - Elastic Spring Force
  - Electric Force

- Examples of Nonconservative forces
  - Friction
  - Air resistance
  - Tension
  - Normal force
  - Propulsion force of things like rocket engine

- Each of these forces depends on the path

03-02 POTENTIAL ENERGY AND CONSERVATIVE FORCES

- Potential energy can be converted into Kinetic energy and back
- Think of an object thrown up
- Bottom → 0 PE, high KE
- Top → high PE, 0 KE

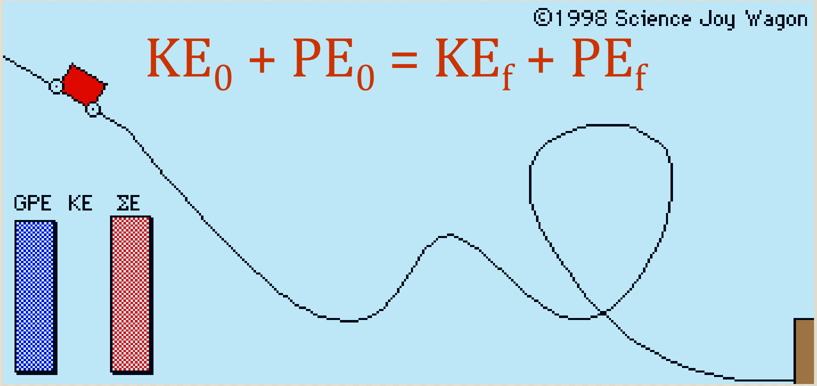
- $\Delta KE = -\Delta PE$
- $KE_f - KE_0 = -(PE_f - PE_0)$
- Rearrange
- Conservation of Mechanical Energy

$$KE_f + PE_f = KE_0 + PE_0$$

  - if only conservative forces do net work

03-02 POTENTIAL ENERGY AND CONSERVATIVE FORCES

- If there is no work done by nonconservative forces
- Total mechanical energy is constant



$$KE_0 + PE_0 = KE_f + PE_f$$

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03-02 POTENTIAL ENERGY AND CONSERVATIVE FORCES

- A toy gun uses a spring to shoot plastic balls ($m = 50\ g$). The spring is compressed by 3.0 cm. Let $k = 2.22 \times 10^5\ N/m$.
- (a) Of course, you have to do some work on the gun to arm it. How much work do you have to do?
- (b) Suppose you fire the gun horizontally. How fast does the ball leave the gun?
- (c) Now suppose you fire the gun straight upward. How high does the ball go?

$$a)\ W = Fd$$
$$W = \frac{1}{2}kx^2$$
$$W = \frac{1}{2}\left(2.22 \times 10^5 \frac{N}{m}\right)(0.03\ m)^2 = 99.9\ J$$

$$b)\ KE_f + PE_f = KE_0 + PE_0$$
$$\frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2$$
$$\frac{1}{2}(0.050\ kg)v_f^2 + 0 = 0 + \frac{1}{2}\left(2.22 \times 10^5 \frac{N}{m}\right)(0.03\ m)^2$$
$$0.025\ kg\ v_f^2 = 99.9\ J$$
$$v_f^2 = 3996 \frac{m^2}{s^2}$$
$$v_f = 63.2 \frac{m}{s}$$
$$c)\ KE_f + PE_{Gf} + PE_{Sf} = KE_0 + PE_{G0} + PE_{S0}$$

At end of barrel

$$\frac{1}{2}mv_f^2 + mgh_f + 0 = 0 + 0 + \frac{1}{2}kx_0^2$$

$$\frac{1}{2}(0.050\ kg)v_f^2 + (0.050\ kg)\left(9.80\frac{m}{s^2}\right)(0.03\ m) = \frac{1}{2}\left(2.22\times 10^5\frac{N}{m}\right)(0.03\ m)^2$$

$$0.025\ kg\ v_f^2 + 0.0147\ J = 99.9\ J$$

$$0.025\ kg\ v_f^2 = 99.8853\ J$$

$$v_f^2 = 3995.412\frac{m^2}{s^2}$$

$$v_f = 63.2\frac{m}{s}$$

At top of path

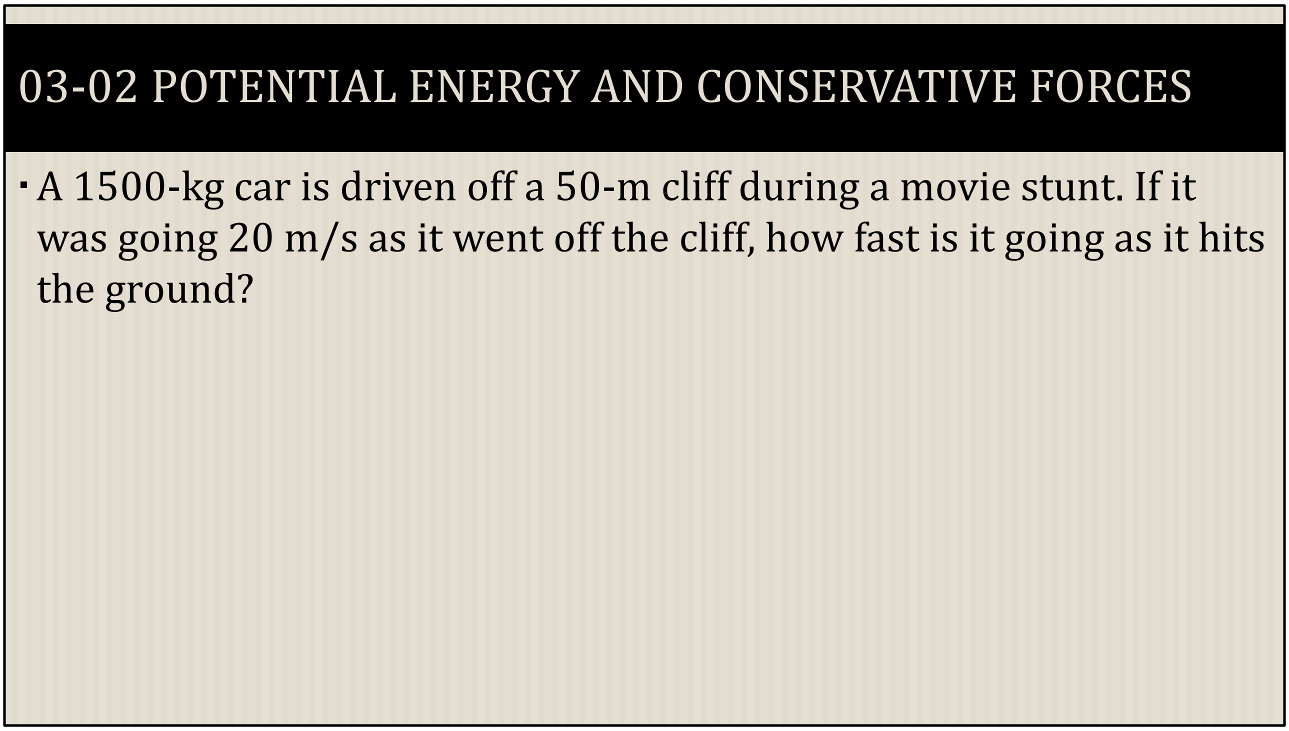
$$KE_f + PE_{Gf} = KE_0 + PE_{G0}$$

$$\frac{1}{2}mv_f^2 + mgh_f = \frac{1}{2}mv_0^2 + mgh_0$$

$$0 + (0.050\ kg)\left(9.80\frac{m}{s^2}\right)h_f = \frac{1}{2}(0.050\ kg)\left(63.2\frac{m}{s}\right)^2 + 0$$

$$\left(0.49\ kg\cdot\frac{m}{s^2}\right)h_f = 99.9\ J$$

$$h_f = 204\ m$$



03-02 POTENTIAL ENERGY AND CONSERVATIVE FORCES

- A 1500-kg car is driven off a 50-m cliff during a movie stunt. If it was going 20 m/s as it went off the cliff, how fast is it going as it hits the ground?

$$PE_0 + KE_0 = PE_f + KE_f$$

$$mgh_0 + \frac{1}{2}mv_0^2 = mgh_f + \frac{1}{2}mv_f^2$$

$$(1500\ kg)\left(9.8\frac{m}{s^2}\right)(50\ m) + \frac{1}{2}(1500\ kg)\left(20\frac{m}{s}\right)^2 = 0 + \frac{1}{2}(1500\ kg)v_f^2$$

$$v_f = 37.1\ m/s$$

03-02 HOMEWORK

- You have great potential…

- Read 7.5, 7.6

03-03 NONCONSERVATIVE FORCES AND CONSERVATION OF ENERGY

In this lesson you will…

- Define nonconservative forces and explain how they affect mechanical energy.
- Show how the principle of conservation of energy can be applied by treating the conservative forces in terms of their potential energies and any nonconservative forces in terms of the work they do.
- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

03-03 NONCONSERVATIVE FORCES AND CONSERVATION OF ENERGY

- Often both conservative and nonconservative forces act on an object at once.
- We can write Work done by net external force as
    - $W = W_c + W_{nc}$
    - $W_c = -\Delta PE, W = \Delta KE$
    - $W_{nc} = \Delta KE + \Delta PE$
    - $KE_0 + PE_0 + W_{nc} = KE_f + PE_f$
    - $E_0 + W_{nc} = E_f$

03-03 NONCONSERVATIVE FORCES AND CONSERVATION OF ENERGY

- Law of Conservation of Energy
  - The total energy is constant in any process. It may change form or be transferred from one system to another, but the total remains the same

- Energy is transformed from one form to another
  - Box sliding down incline
    - PE transformed to KE
    - KE transformed to Heat and Sound
  - Engine
    - Chemical to KE and Heat

03-03 NONCONSERVATIVE FORCES AND CONSERVATION OF ENERGY

- Efficiency
  - Useful energy output is always less than energy input
    - Some energy lost to heat, etc.

$$Efficiency\ (Eff) = \frac{\text{useful energy or work output}}{\text{total energy input}} = \frac{W_{out}}{E_{in}}$$

03-03 NONCONSERVATIVE FORCES AND CONSERVATION OF ENERGY

- A rocket starts on the ground at rest. Its final speed is 500 m/s and height is 5000 m. If the mass of the rocket stays approximately 200 kg. Find the work done by the rocket engine.

- W = 3.48 x 10^7 J

$$E_0 + W_{nc} = E_f$$

$$\frac{1}{2}mv_0^2 + mgh_0 + W_{nc} = \frac{1}{2}mv_f^2 + mgh_f$$

$$\frac{1}{2}(200\ kg)(0)^2 + (200\ kg)\left(9.8\frac{m}{s^2}\right)(0) + W_{nc}$$

$$= \frac{1}{2}(200\ kg)\left(500\frac{m}{s}\right)^2 + (200\ kg)\left(9.8\frac{m}{s^2}\right)(5000\ m)$$

$$W_{nc} = 2.50 \times 10^7\ J + 9.80 \times 10^6\ J$$

$$W_{nc} = 3.48 \times 10^7\ J$$

03-03 NONCONSERVATIVE FORCES AND CONSERVATION OF ENERGY

- A 1500-kg car's brakes failed and it coasts down a hill from rest. The hill is 10 m high and the car has a speed of 12 m/s at the bottom of the hill. How much work did friction do on the car?

- W_f = -39000 J

$$E_0 + W_{nc} = E_f$$
$$\frac{1}{2}mv_0^2 + mgh_0 + W_{nc} = \frac{1}{2}mv_f^2 + mgh_f$$
$$0 + (1500\ kg)\left(9.8\frac{m}{s^2}\right)(10\ m) + W_{nc} = \frac{1}{2}(1500\ kg)\left(12\frac{m}{s}\right)^2 + 0$$
$$W_{nc} = -39000\ J$$

03-03 NONCONSERVATIVE FORCES AND CONSERVATION OF ENERGY

- Captain Proton’s rocket pack provides 800,000 J of work to propel him from resting on his ship which is near the earth to 50 m above it. Captain Proton’s mass is 90 kg. What is his final velocity?
- V = 130 m/s

$$E_0 + W_{nc} = E_f$$

$$\frac{1}{2}mv_0^2 + mgh_0 + W_{nc} = \frac{1}{2}mv_f^2 + mgh_f$$

$$0 + 0 + 800000\ J = \frac{1}{2}(90\ kg)v_f^2 + (90\ kg)\left(9.8\frac{m}{s^2}\right)(50\ m)$$

$$129.6\frac{m}{s} = v_f$$

03-03 HOMEWORK

- Energy is not to be conserved while you do this homework

- Read 7.7

03-04 POWER

In this lesson you will…

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

03-04 POWER

- Two cars with the same mass do the same amount of work to get to 100 km/h.

- Which car is better
  - Takes 8.0 s
  - Takes 6.2 s

- Sometimes the time taken to do the work is important

03-04 POWER

- Rate that work is done

$$P = \frac{W}{t}$$

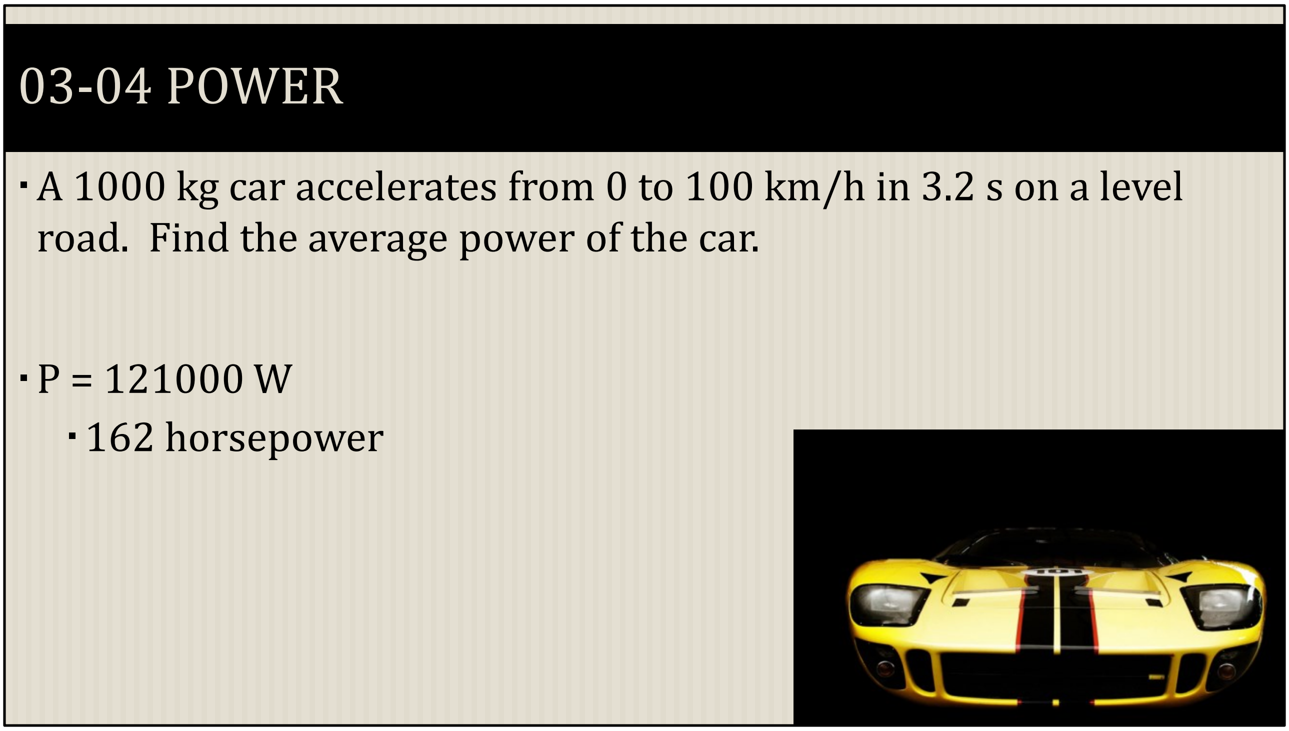
- Unit: joule/s = **watt (W)**

Unit named after James Watt who invented the steam engine
In the American system, horsepower is often used
One horsepower is moving 550 pounds 1 foot in 1 second

03-04 POWER

- Since work changes the amount of energy in an object
- Power is the rate that energy is changing

Power in the human body would be how quickly calories are being burned
Look at the table on page 166 to compare the power with the activity



03-04 POWER

- A 1000 kg car accelerates from 0 to 100 km/h in 3.2 s on a level road. Find the average power of the car.

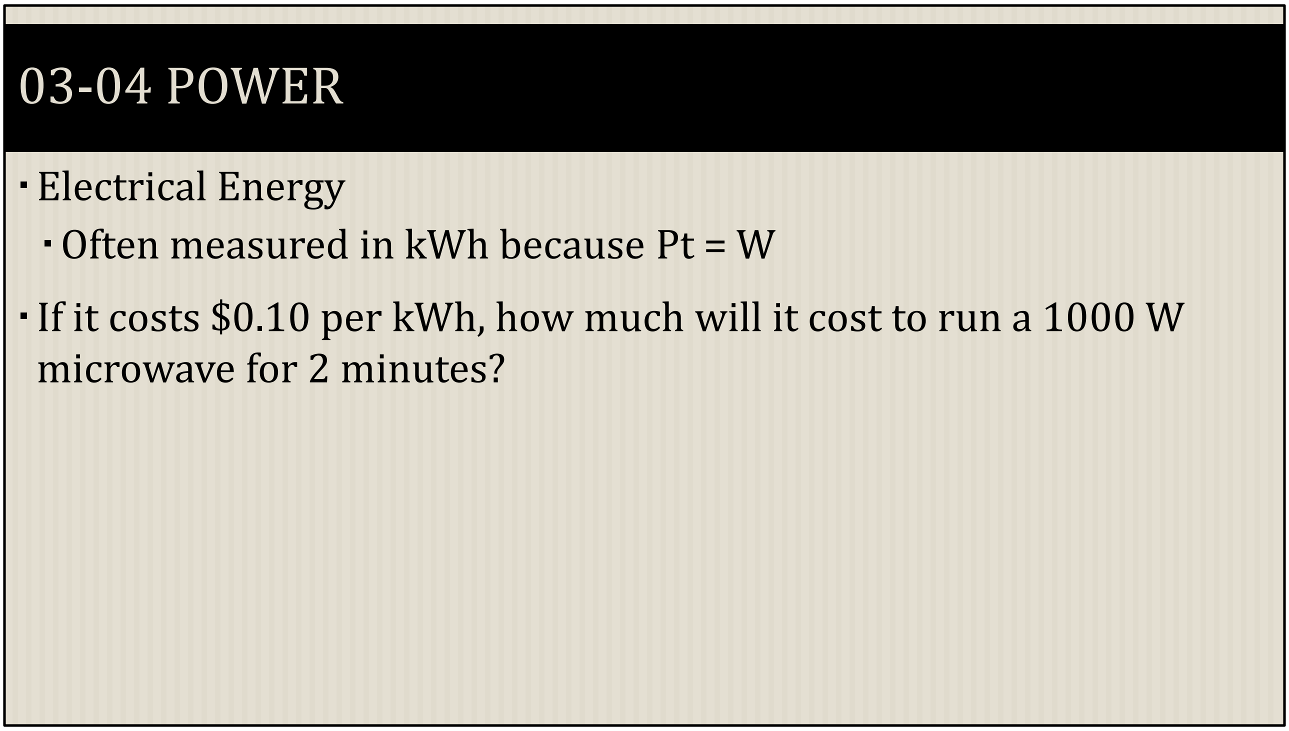
- P = 121000 W
  - 162 horsepower

$v_0 = 0$
$v_f = 100$ km/h = 27.78 m/s
t = 3.2 s
m = 500 kg

$$P = \frac{W}{t}$$

$$P = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2}{t}$$

$$P = \frac{\frac{1}{2}(1000\ kg)\left(27.78\frac{m}{s}\right)^2 - 0}{3.2\ s} = 121000\ W$$



03-04 POWER

- Electrical Energy
  - Often measured in kWh because Pt = W
- If it costs $0.10 per kWh, how much will it cost to run a 1000 W microwave for 2 minutes?

$$P = 1000\ W = 1\ kW, t = 2\ min = \frac{1}{30} h$$

$$P = \frac{W}{t}$$

$$1\ kW = \frac{W}{\frac{1}{30}\ h}$$

$$W = \frac{1}{30}\ kWh$$

$$cost = \frac{1}{30} kWh(\$0.10) = \$0.0033$$

03-04 POWER

Power of Pushups Lab

1. Choose a group member to do pushups. They need to know their weight. Convert their weight to mass in kg.
2. A person lifts approximately 65% of their mass when doing a pushup. How much mass will your group member be lifting?
3. Measure the height of their shoulders at the lowest part of a pushup.
4. Measure the height of their shoulders at the highest part of a pushup.
5. What distance do the shoulders move during a pushup (just going up)?
6. How much work is done for one pushup?
7. How much work is done for 10 pushups?
8. Time how long it takes your group member to do 10 pushups.
9. Calculate the power of doing 10 pushups by your group member.
10. Compare your result with other groups.

03-04 HOMEWORK

- Power through these problems in no time.

- Read 7.8, 7.9

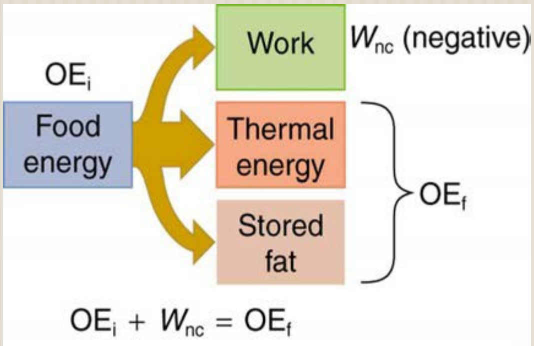
03-05 ENERGY IN HUMANS AND THE WORLD

In this lesson you will…

- Explain the human body's consumption of energy when at rest vs. when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.
- Describe the distinction between renewable and nonrenewable energy sources.
- Explain why the inevitable conversion of energy to less useful forms makes it necessary to conserve energy resources.

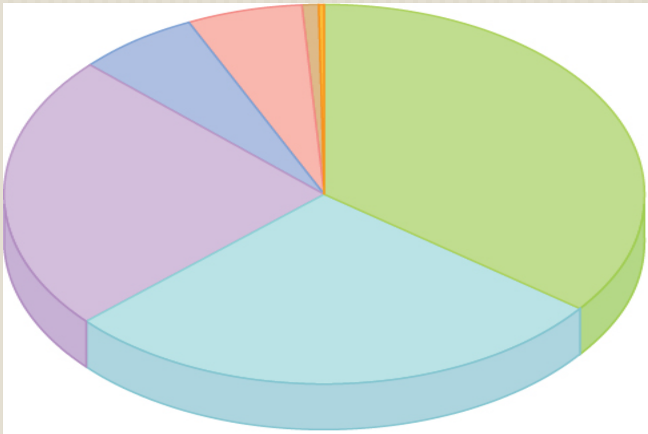
03-05 ENERGY IN HUMANS AND THE WORLD

- Human bodies (all living bodies) convert energy
- Rate of food energy use is metabolic rate
- Basal metabolic rate (BMR)
  - Total energy conversion at rest
  - Highest: liver and spleen
  - See table 7.4
- Table 7.5 shows energy consumed for various activites



03-05 ENERGY IN HUMANS AND THE WORLD

- Energy is required to do work
- World wide, the most common source of energy is oil



🟩	Petroleum:	3527 ~ 35.43%
🟦	Coal:	2802 ~ 28.15%
🟪	Dry natural gas:	2335 ~ 23.46%
🟦	Hydro-electricity:	624 ~ 6.27%
🟥	Nuclear-electricity:	576 ~ 5.79%
🟫	Geothermal, wind, solar, biomass:	86 ~ 0.86%
🟧	Geothermal, biomass, solar not used for electricity:	5 ~ 0.05%

Total: 9955

From 2008 units are billions of kWh

03-05 ENERGY IN HUMANS AND THE WORLD

- USA has 4.5% of world population, but uses 24% of world’s oil

- World energy consumption continues to increase quickly
  - Growing economies in China and India
  - Fossil Fuels are very polluting
  - Many countries trying to develop renewable energy like wind and solar
- Generally, higher energy use per capita = better standard of living

03-05 ENERGY IN HUMANS AND THE

Ludington Pumped Storage Power Plant

- It consists of a reservoir 110 feet (34 m) deep, 2.5 miles (4.0 km) long, and one mile (1.6 km) wide which holds 27 billion US gallons (100 Gl) of water. The 1.3-square-mile (3.4 km2) reservoir is located on the banks of Lake Michigan.
- The power plant consists of six reversible turbines that can each generate 312 megawatts of electricity for a total output of 1,872 megawatts.
- At night, during low demand for electricity, the turbines run in reverse to pump water 363 feet (111 m) uphill from Lake Michigan into the reservoir.
- During periods of peak demand water is released to generate power. Electrical generation can begin within two minutes with peak electric output of 1872 MW achieved in under 30 minutes. Maximum water flow is over 33 million US gallons (120,000 m^3) per minute.
- This process was designed to level the load of nearby nuclear power plants on the grid. It also replaces the need to build natural gas peak power plants used only during high demand.

03-05 HOMEWORK

- You have the power to change to world, but will you work to do it?

- Read 8.1, 8.2

03-06 IMPULSE AND MOMENTUM

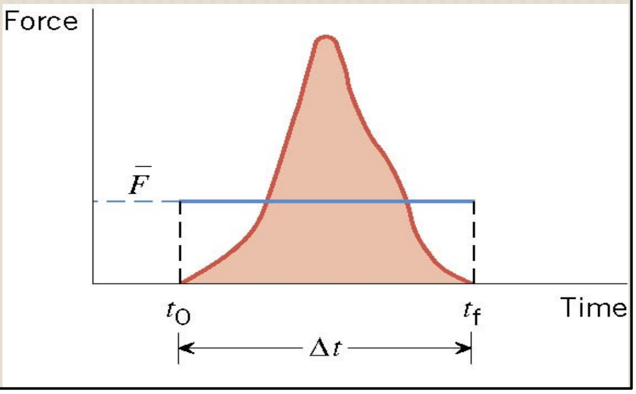
In this lesson you will…

- Define linear momentum.
- Explain the relationship between momentum and force.
- Calculate momentum given mass and velocity.
- Define impulse.
- Describe effects of impulses in everyday life.
- Determine the average effective force using graphical representation.
- Calculate average force and impulse given mass, velocity, and time.

03-06 IMPULSE AND MOMENTUM

- Often the force acting on an object is not constant.
  - Baseball or Tennis ball being hit

- Times of force often short
- Force can be huge



03-06 IMPULSE AND MOMENTUM

- To hit a ball well
  - Both size of force and time of contact are important
  - Bring both these together in concept of impulse

03-06 IMPULSE AND MOMENTUM

- Impulse

$$J = F\Delta t$$

- Unit: Ns

- Is a vector

03-06 IMPULSE AND MOMENTUM

- Object responds to amount of impulse
- Large impulse → Large response → higher v_f
- Large mass → less velocity
- Both mass and velocity play role in how responds to impulse

03-06 IMPULSE AND MOMENTUM

- Linear Momentum

$$p = mv$$

- Unit: kg m/s
- Is a vector
- Is important when talking about collisions

03-06 IMPULSE AND MOMENTUM

$$F = ma$$

$$a = \frac{v_f - v_0}{\Delta t}$$

$$F = m\frac{v_f - v_0}{\Delta t} = \frac{mv_f - mv_0}{\Delta t}$$

$$F\Delta t = mv_f - mv_0$$

03-06 IMPULSE AND MOMENTUM

- Impulse = Change in Momentum
- Hard to measure force during contact
- Find change in momentum
  - Use impulse-Momentum Theorem and time of contact to find average force of contact
- Watch NASCAR Crash

03-06 IMPULSE AND MOMENTUM

- A baseball (m = 0.14 kg) with initial velocity of -40 m/s (90 mph) is hit. It leaves the bat with a velocity of 60 m/s after 0.001 s. What is the impulse and average net force applied to the ball by the bat?

- Impulse = 14 Ns
- F = 14000 N

$$F\Delta t = mv_f - mv_0$$

$$F\Delta t = (0.14\ kg)\left(60\frac{m}{s}\right) - (0.14kg)\left(-40\frac{m}{s}\right) = 14\ kg\ m/s$$

$$F = \frac{14\ kg\frac{m}{s}}{0.001\ s} = 14000\ N$$

03-06 IMPULSE AND MOMENTUM

- A raindrop (m = .001 kg) hits a roof of a car at -15 m/s. After it hits, it spatters so the effective final velocity is 0. The time of impact is .01 s. What is the average force?
  - F = 1.5 N
- What if it is ice so that it bounces off at 10 m/s?
  - F = 2.5 N
- Watch Offset Crash

$$F\Delta t = mv_f - mv_0$$

$$F(0.01\ s) = (0.001\ kg)0 - (0.001\ kg)\left(-15\frac{m}{s}\right)$$

$$F(0.01\ s) = 0.015\ kg\frac{m}{s}$$

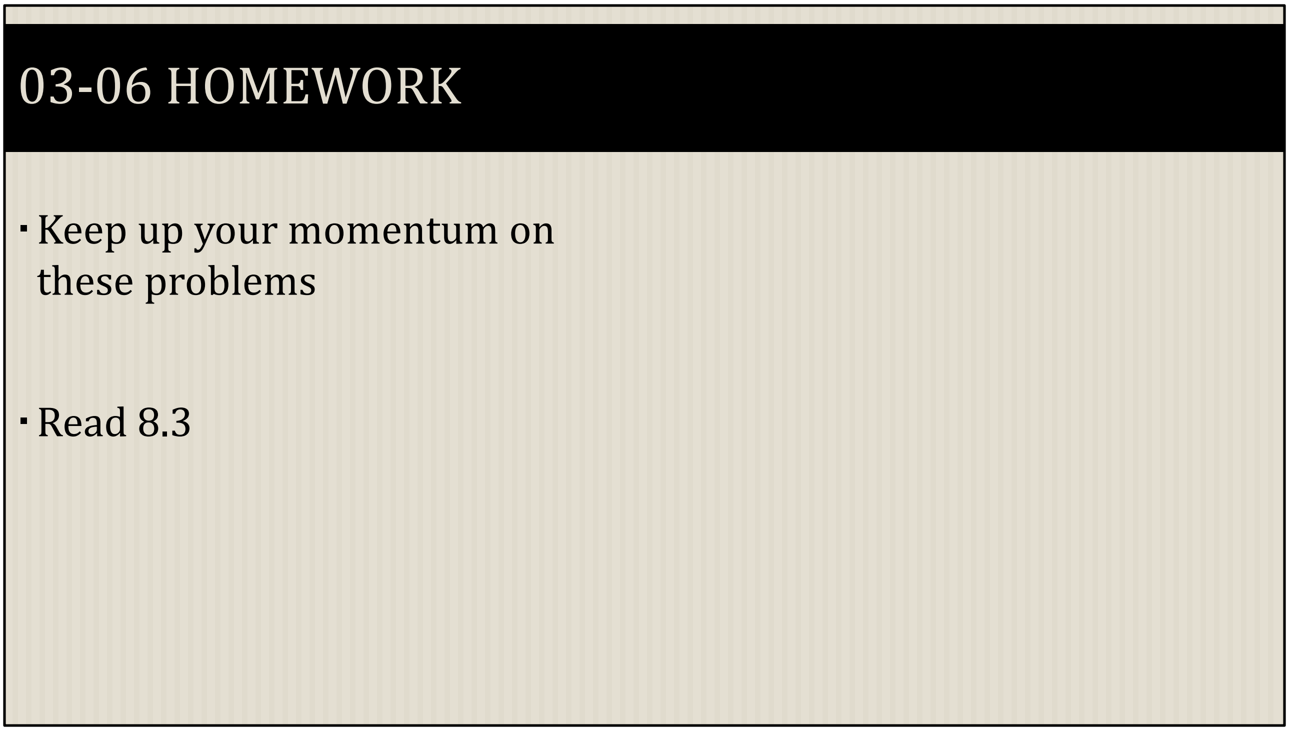
$$F = 1.5\ N$$

$$F(0.01\ s) = (0.001\ kg)\left(10\frac{m}{s}\right) - (0.001\ kg)\left(-15\frac{m}{s}\right)$$

$$F(0.01\ s) = 0.025\ kg\frac{m}{s}$$

$$F = 2.5\ N$$

Hailstones are usually more massive than raindrops so that the force is even greater.
The rebounding adds force to the collision



03-06 HOMEWORK

- Keep up your momentum on these problems
- Read 8.3

03-07 CONSERVATION OF MOMENTUM

In this lesson you will…

- Describe the principle of conservation of momentum.
- Derive an expression for the conservation of momentum.
- Explain conservation of momentum with examples.

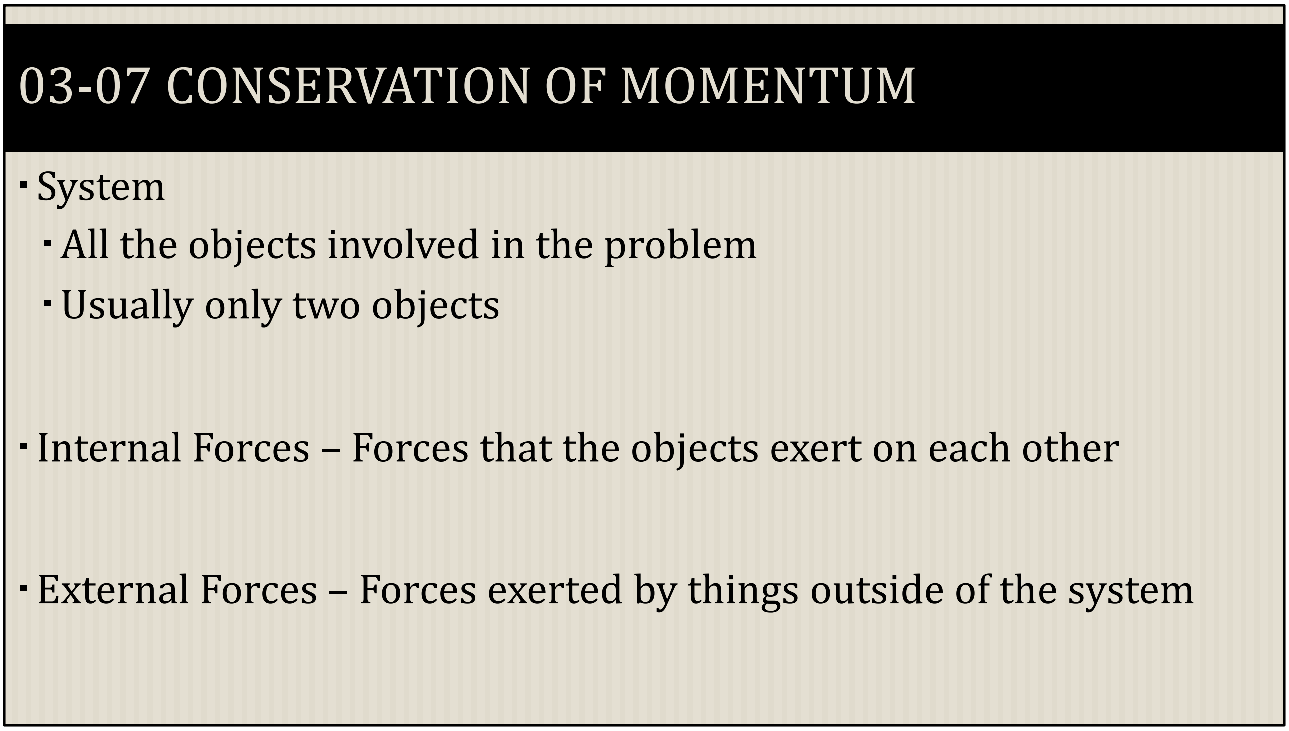
03-06B BUMPER TESTING LAB

1. Each team makes a bumper out of paper and tape.
   a. 2.5 cm × 4 cm × 10 cm
   b. Do not use excessive tape
2. The bumper is placed against the end of the track.
3. The cart is released from a distance as set by the teacher.
4. The maximum force is read from the sensor.

03-07 CONSERVATION OF MOMENTUM

- Do the lab in your worksheet.
- Explain why if a person standing of frictionless ice shoots a bullet at 200 m/s does not move backwards are 200 m/s.
- A 100 kg person pushes off from a 50 kg person on frictionless ice. If the 100 kg person moves at 3 m/s, what speed will the 50 kg person move at?

The person has more mass than the bullet.
Since they are half the mass, they will move at twice the speed. Mass and speed ratios are reciprocals.



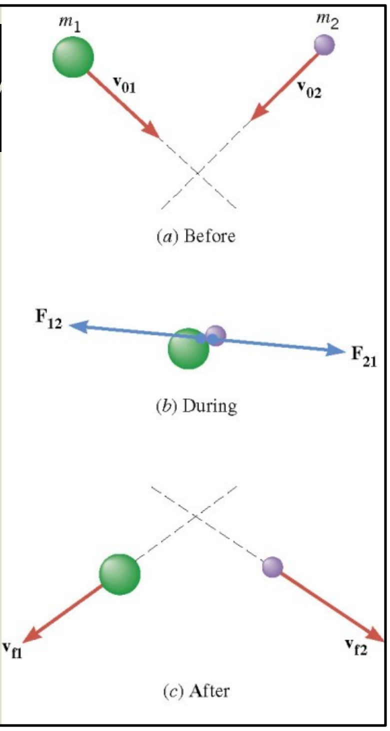
03-07 CONSERVATION OF MOMENTUM

- System
  - All the objects involved in the problem
  - Usually only two objects
- Internal Forces – Forces that the objects exert on each other
- External Forces – Forces exerted by things outside of the system

Usually only two objects for linear momentum because very rarely do more than two object hit at the same time. It usually happens that two hit, and then one of those hits another.
Internal forces → the objects pushing each other
External forces → gravity pulling the objects down

03-07 CONSERVATION OF MOMENTUM

- Two balls hit in the air
- During the collision
  - Internal Forces = F_{12} and F_{21}
  - External Forces = Weight (W_1 and W_2)



F_{12} and F_{21} are action-reaction pair from Newton’s Third Law are equal and opposite

03-07 CONSERVATION OF MOMENTUM

- $F\Delta t = mv_f - mv_0$
- Object 1: $(W_1 + F_{12})\Delta \mathrm{t} = \mathrm{m_1 v_{f1} - m_1 v_{01}}$
- Object 2: $(W_2 + F_{21})\Delta \mathrm{t} = \mathrm{m_2 v_{f2} - m_2 v_{02}}$
- Add
- $(W_1 + W_2 + F_{12} + F_{21})\Delta \mathrm{t} = (\mathrm{m_1 v_{f1} + m_2 v_{f2}}) - (\mathrm{m_1 v_{01} + m_2 v_{02}})$

- $(Ext\ F + Int\ F)\Delta t = p_f - p_0$

03-07 CONSERVATION OF MOMENTUM

- Since F_{12} and F_{21} are equal and opposite
  - Sum of internal forces = 0
- (External Forces)$\Delta t = p_f - p_0$
- If Isolated system:
- $0 = p_f - p_0$ OR $p_0 = p_f$

Isolated system = no external forces

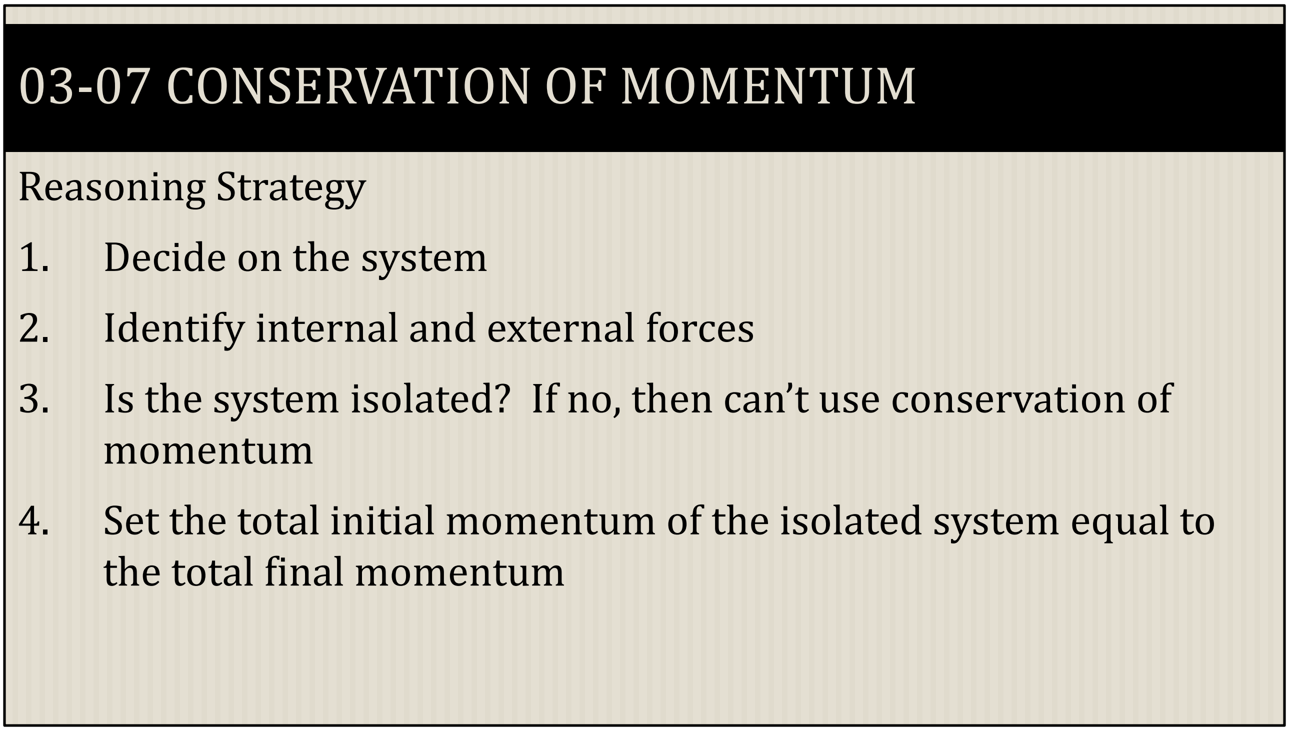
03-07 CONSERVATION OF MOMENTUM

- Law of Conservation of Momentum
  - In an isolated system the total momentum remains constant

$$p_0 = p_f$$

  - System can contain any number of objects

- Watch Crash Video



03-07 CONSERVATION OF MOMENTUM

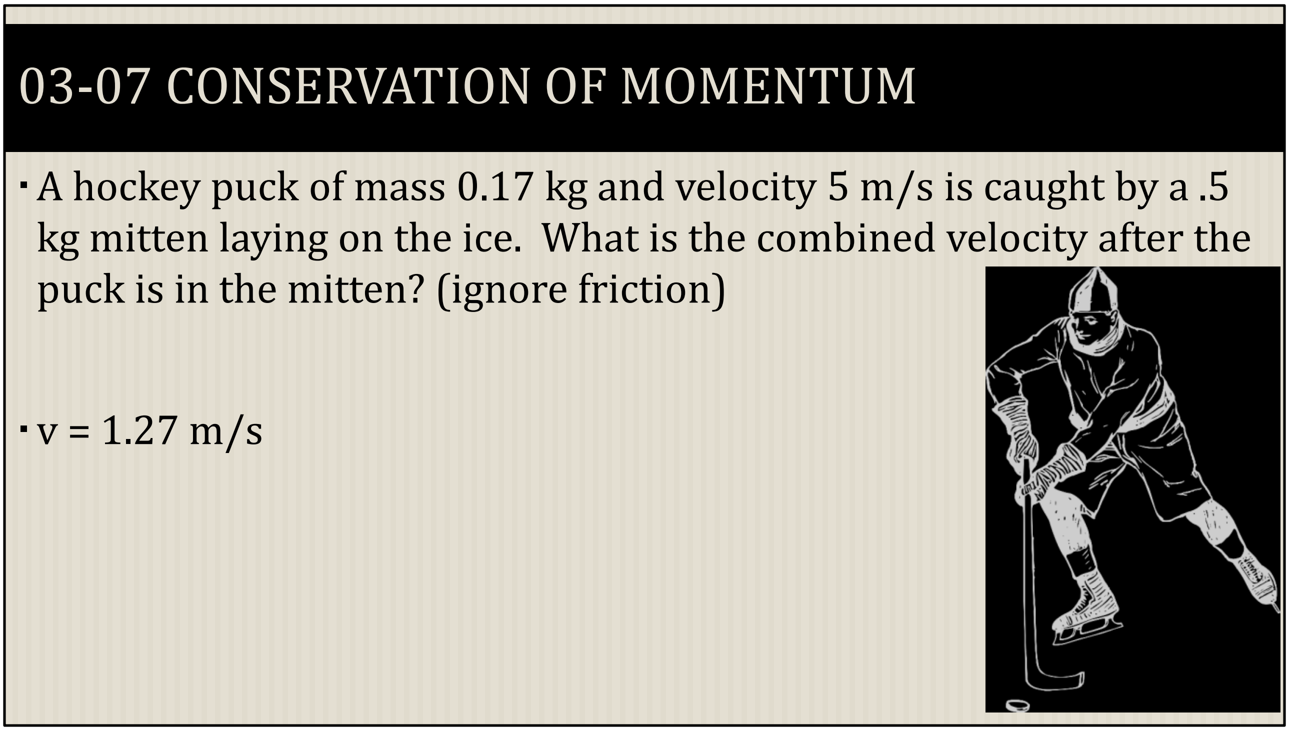
Reasoning Strategy

1. Decide on the system
2. Identify internal and external forces
3. Is the system isolated? If no, then can't use conservation of momentum
4. Set the total initial momentum of the isolated system equal to the total final momentum

03-07 CONSERVATION OF MOMENTUM

- Two billiard balls are colliding on a table. In order to apply the law of conservation of momentum, what should the system be? One ball or both billiard balls?
  - Two billiard balls.

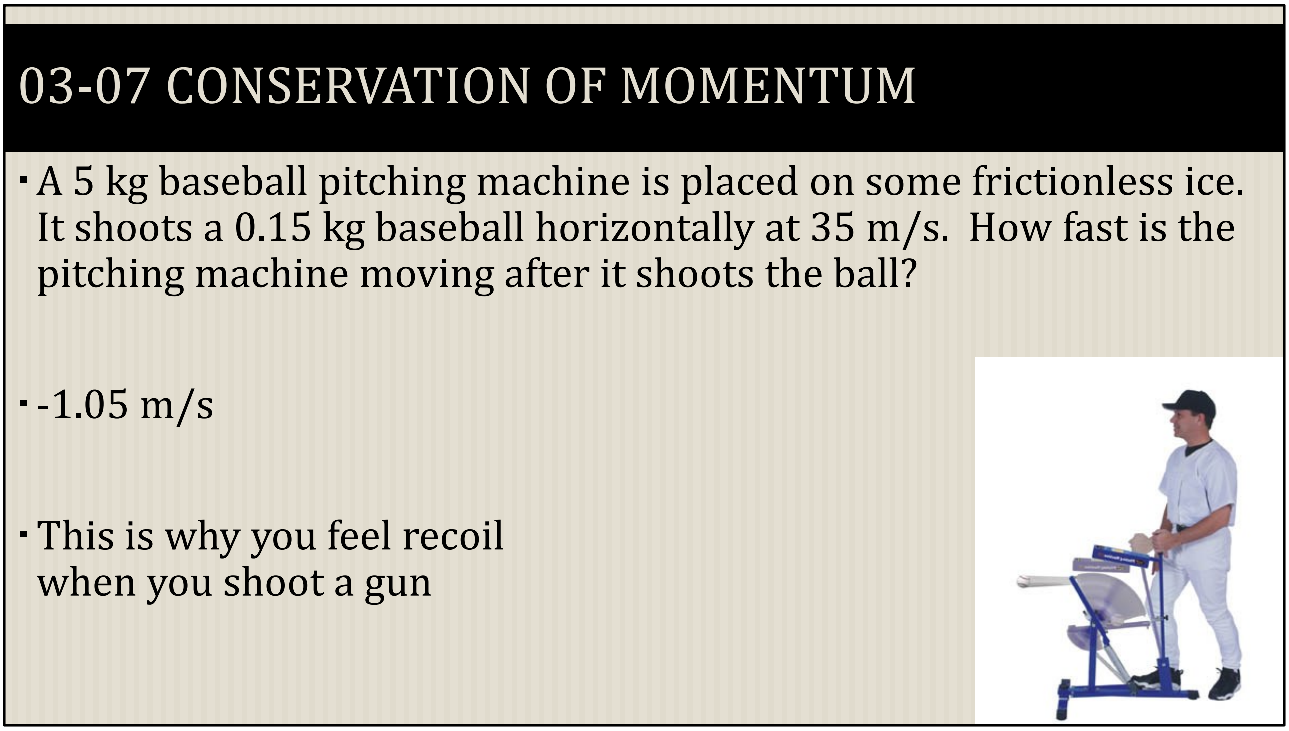
- External Forces: Weight and Normal Force
  - If the table is horizontal these cancel
- If it were one ball, then the force of the second ball hitting it would not cancel with anything.



03-07 CONSERVATION OF MOMENTUM

- A hockey puck of mass 0.17 kg and velocity 5 m/s is caught by a .5 kg mitten laying on the ice. What is the combined velocity after the puck is in the mitten? (ignore friction)
- v = 1.27 m/s

$$p_0 = p_f$$
$$m_1 v_{01} + m_2 v_{02} = m_1 v_{f1} + m_2 v_{f2}$$
$$(0.17\ kg)\left(5\ \frac{m}{s}\right) + (0.5\ kg)\left(0\frac{m}{s}\right) = (0.17\ kg)v + (0.5\ kg)v$$
$$0.85\ kg\frac{m}{s} = (0.67\ kg)v$$
$$v = 1.27\ m/s$$



03-07 CONSERVATION OF MOMENTUM

- A 5 kg baseball pitching machine is placed on some frictionless ice. It shoots a 0.15 kg baseball horizontally at 35 m/s. How fast is the pitching machine moving after it shoots the ball?
- -1.05 m/s
- This is why you feel recoil when you shoot a gun

$$
\begin{gathered}
p_0 = p_f \\
m_1 v_{01} + m_2 v_{02} = m_1 v_{f1} + m_2 v_{f2} \\
(5\ kg)(0) + (0.15\ kg)(0) = (5\ kg)v + (0.15kg)(35\ m/s) \\
0 = (5\ kg)v + 5.25\ kg\ m/s \\
-(5\ kg)v = -5.25\ kg\ m/s \\
v = 1.05\ m/s
\end{gathered}
$$

03-07 HOMEWORK

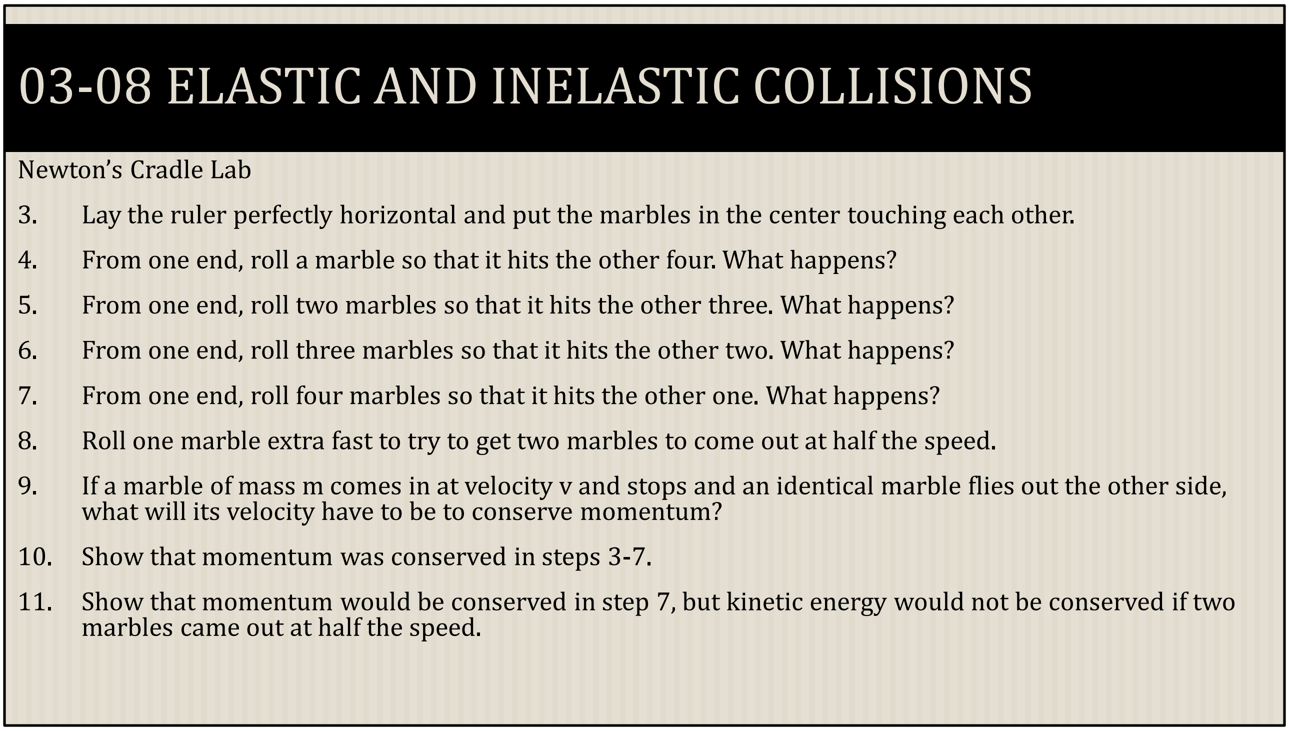
- Solving problems is fun!

- Read 8.4, 8.5

03-08 ELASTIC AND INELASTIC COLLISIONS

In this lesson you will…

- Describe an elastic collision of two objects in one dimension.
- Determine the final velocities in an elastic collision given masses and initial velocities.
- Define inelastic collision.
- Explain perfectly inelastic collision.



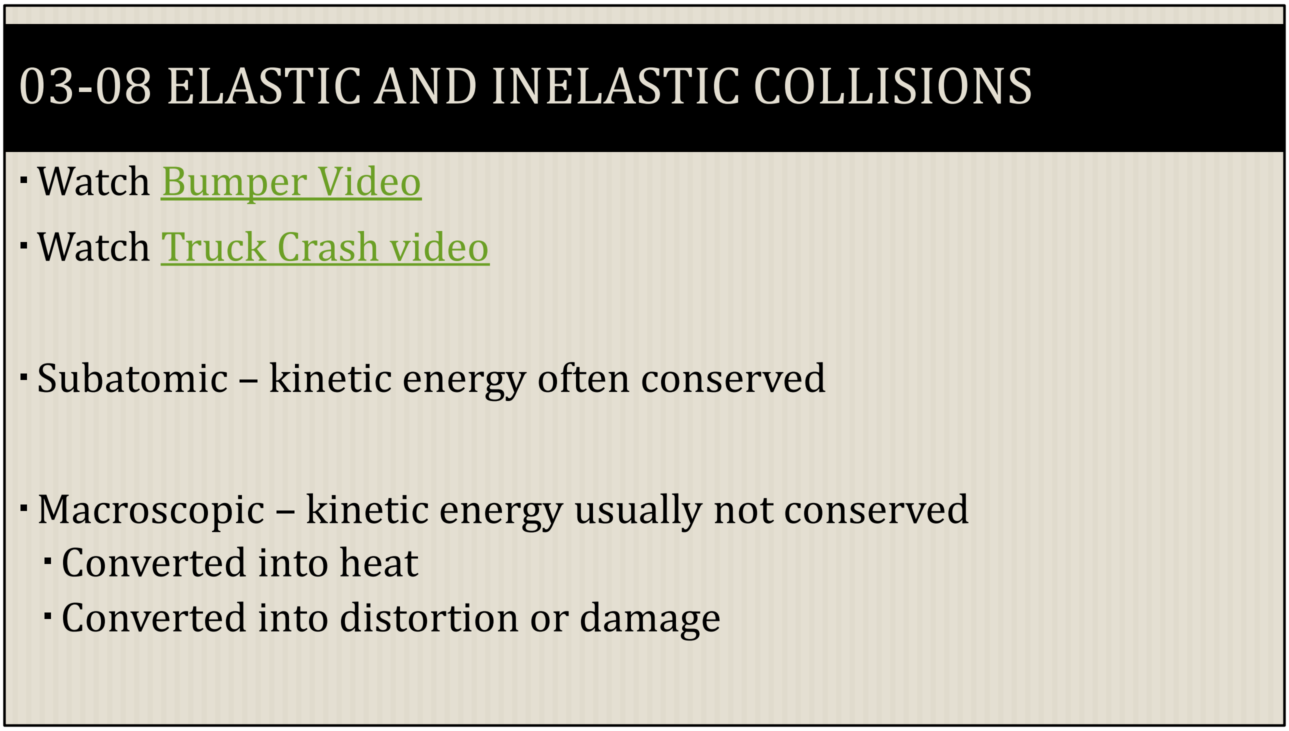
03-08 ELASTIC AND INELASTIC COLLISIONS

Newton’s Cradle Lab

3. Lay the ruler perfectly horizontal and put the marbles in the center touching each other.
4. From one end, roll a marble so that it hits the other four. What happens?
5. From one end, roll two marbles so that it hits the other three. What happens?
6. From one end, roll three marbles so that it hits the other two. What happens?
7. From one end, roll four marbles so that it hits the other one. What happens?
8. Roll one marble extra fast to try to get two marbles to come out at half the speed.
9. If a marble of mass m comes in at velocity v and stops and an identical marble flies out the other side, what will its velocity have to be to conserve momentum?
10. Show that momentum was conserved in steps 3-7.
11. Show that momentum would be conserved in step 7, but kinetic energy would not be conserved if two marbles came out at half the speed.

4. One flies out
5. Two fly out
6. Three fly out
7. Four fly out
8. Can’t
9. *v*
10. *mv* is the same before and after
11. *mv* is the same before and after, but $KE = \frac{1}{2}mv^2$ will not be the same because of the v^2

$$KE = KE$$
$$\frac{1}{2}m(2v)^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$
$$2mv^2 \neq mv^2$$



03-08 ELASTIC AND INELASTIC COLLISIONS

- Watch Bumper Video
- Watch Truck Crash video

- Subatomic – kinetic energy often conserved

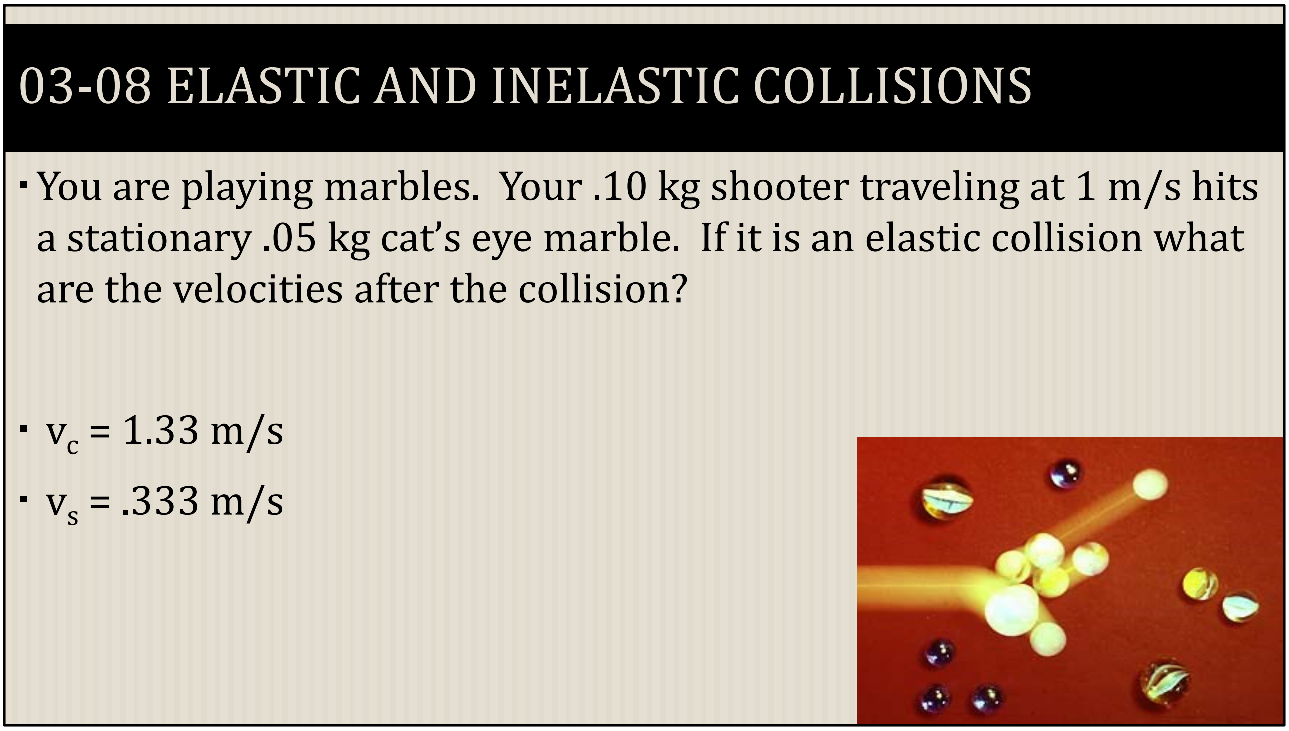
- Macroscopic – kinetic energy usually not conserved
  - Converted into heat
  - Converted into distortion or damage

Demo energy lost to heat by smashing two steel balls together

03-08 ELASTIC AND INELASTIC COLLISIONS

- Elastic – kinetic energy conserved
- Inelastic – kinetic energy not conserved
- Completely inelastic – the objects stick together

SUV crash test video
NASCAR video
Crash test humor



03-08 ELASTIC AND INELASTIC COLLISIONS

- You are playing marbles. Your .10 kg shooter traveling at 1 m/s hits a stationary .05 kg cat's eye marble. If it is an elastic collision what are the velocities after the collision?

- $v_c = 1.33$ m/s
- $v_s = .333$ m/s

Momentum

$$m_s v_{0s} + m_c v_{0c} = m_s v_{fs} + m_c v_{fc}$$
$$(0.1\ kg)\left(1\frac{m}{s}\right) + (0.05\ kg)(0) = (0.1\ kg)v_{fs} + (0.05\ kg)v_{fc}$$
$$0.1\ kg\frac{m}{s} = (0.1\ kg)v_{fs} + (0.05\ kg)v_{fc}$$
$$v_{fs} = 1\ m/s - 0.5\ v_{fc}$$

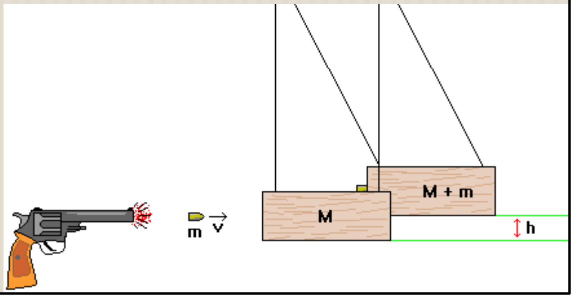
Kinetic Energy

$$\frac{1}{2}m_s v_{0s}^2 + \frac{1}{2}m_c v_{0c}^2 = \frac{1}{2}m_s v_{fs}^2 + \frac{1}{2}m_c v_{fc}^2$$
$$\frac{1}{2}(0.1\ kg)\left(1\frac{m}{s}\right)^2 + 0 = \frac{1}{2}(0.1\ kg)v_{fs}^2 + \frac{1}{2}(0.05\ kg)v_{fc}^2$$
$$0.05\ J = (0.05\ kg)v_{fs}^2 + (0.025\ kg)v_{fc}^2$$
$$v_{fs}^2 + 0.5\ v_{fc}^2 = 1\ \left(\frac{m}{s}\right)^2$$
$$\left(1\frac{m}{s} - 0.5v_{fc}\right)^2 + 0.5\ v_{fc}^2 = 1\ \left(\frac{m}{s}\right)^2$$
$$1\ \left(\frac{m}{s}\right)^2 - \left(1\frac{m}{s}\right)v_{fc} + 0.25\ v_{fc}^2 + 0.5\ v_{fc}^2 = 1\ \left(\frac{m}{s}\right)^2$$
$$-\left(1\frac{m}{s}\right)v_{fc} + 0.75\ v_{fc}^2 = 0$$

$$v_{fc}\left(-1\frac{m}{s}+0.75\,v_{fc}\right)=0$$
$$v_{fc}=0\ or\ 1.33\ m/s$$
$$v_{fs}=1\frac{m}{s}-0.5\,v_{fc}$$
$$V_{fs}=1\ m/s-0.5(1.33\ m/s)=0.333\ m/s$$

03-08 ELASTIC AND INELASTIC COLLISIONS

- A ballistic pendulum can be used to determine the muzzle velocity of a gun. A .01 kg bullet is fired into a 3 kg block of wood. The block is attached with a thin .5m wire and swings to an angle of 40°. How fast was the bullet traveling when it left the gun?
- v = 455 m/s



Do an actual demonstration

Collision

$$m_b v_{0b} + m_w v_{0w} = m_b v_{fb} + m_w v_{fw}$$
$$(0.01\ kg)v_{0b} + 0 = (0.01\ kg)v_f + (3\ kg)v_f$$
$$(0.01\ kg)v_{0b} = (3.01\ kg)v_f$$

After collision

$$h = 0.5\ m - 0.5\ m\cos 40° = 0.1170\ m$$
$$PE_0 + KE_0 = PE_f + KE_f$$
$$0 + \frac{1}{2}(3.01\ kg)v_f^2 = (3.01\ kg)\left(9.8\frac{m}{s^2}\right)(0.1170\ m) + 0$$
$$1.505\ kg\ v_f^2 = 3.45\ J$$
$$v_f = 1.51\ m/s$$

Combine

$$(0.01\ kg)v_{0b} = (3.01\ kg)\left(1.51\frac{m}{s}\right)$$
$$v_{0b} = 455\ m/s$$

03-08 ELASTIC AND INELASTIC COLLISIONS

- Watch Child Seat video

- Watch Reducing Risk video

03-08 HOMEWORK

- Bounce through these problems and let the concepts stick to you.